quæ erunt Æquationis datæ Radices omnes reales; hæ nempe ad dextram erunt Radices affirmativæ, illæ verò ad sinistram Radices negativæ. Demonstratio est manifesta ex præcedentibus, habita tantùm ratione Parabolæ per puncta B, C, c, $\varkappa$, K transeuntis. Nam posito F foco Parobolæ, (cujus distantia à Vertice ast $\frac{1}{4}$ ON,) notum est quod lineæ omnes ut FB + BQ, FC + CD, &c, eandem ubique conficiant summam.

Atque ex principiis hic positis proclive erit Instrumentum haud inconcinnum & quantumvis accuratum fabricari, cujus beneficio hujusmodi Æquationum quarumcunque Radices nullo fere negotio inveniri possint, & præ oculis exhiberi. Hoc autem quilibet, si id Curæ sit, variis modis pro ingenio suo efficere potest, & de his jam satis.

III. *Æquationum quarundam Potestatis tertiæ, quintæ, septimæ, nonæ, & superiorum, ad infinitum usque pergendo, in terminis finitis, ad instar Regularum pro Cubicis quæ vocantur* Cardani, *Resolutio Analytica.*

Per Ab. De Moivre, R. S. S.

Sit n Numerus quicunque, y quantitas incognita, sive Æquationis Radix quæsita, sitque a quantitas quævis omnino cognita, sive ut vocant Homogeneum Comparationis: Atque horum inter se relatio exprimatur per Æquationem

$$ny + \frac{nn-1}{2 \times 3} ny^3 + \frac{nn-1}{2 \times 3} \times \frac{nn-9}{4 \times 5} ny^5 + \frac{nn-1}{2 \times 3} \times \frac{nn-9}{4 \times 5} \times \frac{nn-25}{6 \times 7} ny^7, \&c. = a$$

Ex

Ex hujus ſeriei natura manifeſtum eſt, quod ſi n ſumatur numerus aliquis impar (integer ſcilicet, nec refert utrum ſit affirmativus vel negativus) tunc ſeries ſponte ſua terminabitur, & Æquatio fit una ex ſupra præfinitis, cujus Radix eſt

$$(1)\quad y = \tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}+a} - \frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}+a}}$$

$$\text{vel } (2)\quad y = \tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}+a} - \tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}-a}$$

$$\text{vel } (3)\quad y = \frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}-a}} - \tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}-a}$$

$$\text{vel } (4)\quad y = \frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}-a}} - \frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}-a}}$$

Exempli gratia, ſit hujus Æquationis poteſtatis quintæ $5y + 20y^3 + 16y^5 = 4$ Radix invenienda, quo in caſu erit $n = 5$ & $a = 4$. Radix juxta formam primam erit $y = \frac{1}{2}\sqrt[5]{\sqrt{17}+4} - \dfrac{\frac{1}{2}}{\sqrt[5]{\sqrt{17}+4}}$, quæ in numeris vulgaribus expeditiſſime explicari poteſt ad hunc modum. Eſt $\sqrt{17}+4 = 8.1231$, cujus Logarithmus 0. 9097164, & hujus pars quinta 0. 1819433, huic reſpondens numerus eſt $1.5203 = \sqrt[5]{\sqrt{17}+4}$. Ipſius vero 0. 1819433 Complementum Arithmeticum eſt 9. 8180567. cui reſpondet numerus $0.6577 = \dfrac{1}{\sqrt[5]{\sqrt{17}+4}}$ Igitur horum numerorum ſemidifferentia $0.4313 = y$.

Hic venit Observandum quod loco Radicis generalis, non incommode sumeretur $y = \frac{1}{2}\sqrt[n]{2a} - \frac{\frac{1}{2}}{\sqrt[n]{2a}}$, si quando numerus a respectu unitatis, si satis magnus, ut si Æquatio fuerit $5y + 20y^3 + 16y^5 = 682$, erit Log. $2a = 3.1348143$, cujus pars quinta 0.6269628, & huic respondens numerus 4.236. Complementi autem Arithmetici 9.3730372 numerus est 0.236 & horum numerorum semidifferentia $2 = y$.

Atqui præterea, si in Æquatione præcedenti signa alternatim sint affirmantia & negantia, vel quod eodem redit, si series obvenerit hujus modi

$$ny + \frac{1-nn}{2 \times 3}ny^3 + \frac{1-nn}{2 \times 3} \times \frac{9-nn}{4 \times 5}ny^5 + \frac{1-nn}{2 \times 3} \times \frac{9-nn}{4 \times 5} \times \frac{25-nn}{6 \times 7}ny^7, \text{ \&c.} = a$$

erit hujus Radix

$$(1) \quad y = \tfrac{1}{2}\sqrt[n]{a + \sqrt{aa-1}} + \frac{\frac{1}{2}}{\sqrt[n]{a + \sqrt{aa-1}}}$$

$$\text{vel } (2) \quad y = \tfrac{1}{2}\sqrt[n]{a + \sqrt{aa-1}} + \tfrac{1}{2}\sqrt[n]{a - \sqrt{aa-1}}$$

$$\text{vel } (3) \quad y = \frac{\frac{1}{2}}{\sqrt[n]{a - \sqrt{aa-1}}} + \tfrac{1}{2}\sqrt[n]{a - \sqrt{aa-1}}$$

$$\text{vel } (4) \quad y = \frac{\frac{1}{2}}{\sqrt[n]{a - \sqrt{aa-1}}} + \frac{\frac{1}{2}}{\sqrt[n]{a + \sqrt{aa-1}}}$$

Hic autem Notandum, quod si $\frac{n-1}{2}$ numerus extiterit impar, Radicis inventæ signum in ei contrarium permutandum est.

Pro-

Proponatur Æquatio $5y - 20y^3 + 16y^5 = 6$, unde $n = 5$ & $a = 6$. Erit Radix $= \frac{1}{2}\sqrt[5]{6+\sqrt{35}} + \dfrac{\frac{1}{2}}{\sqrt[5]{6+\sqrt{35}}}$

Vel quoniam $6 + \sqrt{35} = 11.916$, erit hujus logarithmus 1.0761304 & ejus pars quinta 0.2152561, Complementum vero Arithmeticum 9.7847439. Horum Logarithmorum numeri sunt 1.6415 & 0.6091 respective, quorum semisumma $1.1253 = y$.

Verum si acciderit ut a sit minor unitate, tunc Radicis forma secunda, ut quæ proposito est magis conveniens, præ reliquis seligenda est. Sic si Æquatio fuerit $5y - 20y^3 + 16y^5 = \frac{61}{64}$, erit $y = \frac{1}{2}\sqrt[5]{\frac{61}{64} + \sqrt{\frac{-375}{4096}}} + \frac{1}{2}\sqrt[5]{\frac{61}{64} - \sqrt{\frac{-375}{4096}}}$. Et quidem si Binomialium Radix quintana ullo pacto extrahi queat, prodibit Radix proba & possibilis, etsi expressio ipsa impossibilitatem mentiatur. Binomialis vero $\frac{61}{64} + \sqrt{\frac{-375}{4096}}$ Radix quintana est $\frac{1}{4} + \frac{1}{4}\sqrt{-15}$, & Binomialis $\frac{61}{64} - \sqrt{\frac{-375}{4096}}$ Radix itidem quintana est $\frac{1}{4} - \frac{1}{4}\sqrt{-15}$, quorum Binomialium semisumma $= \frac{1}{4} = y$.

Si autem extractio ista vel non peragi posset, vel etiam difficilior videretur, res ubique confici potest per Tabulam sinuum naturalium ad modum sequentem.

Ad Radium 1 sit $a = \frac{61}{64} = 0.95112$ sinus arcus cujusdam, qui proinde erit 72°: 23′ cujus pars quinta (eo quod $n = 5$) est 14°: 28′; hujus sinus $0.24981 = \frac{1}{4}$ proxime. Nec secus procedendum in Æquationibus graduum superiorum.